GASpAR Development

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1 Historical and scientific context

Accurate and efficient simulation of strongly turbulent flows is a prevalent challenge in many atmospheric, oceanic, and astrophysical applications. New simulation codes are needed to investigate such flows in the parameter regimes that interest the geophysics communities. Turbulent flows are linked to many issues in the geosciences, for example, in meteorology, oceanography, climatology, ecology, solar-terrestrial interactions, and solar fusion, as well as dynamo effects, specifically, magnetic-field generation in cosmic bodies by turbulent motions. Nonlinearities prevail when the Reynolds number Re (the ratio of the nonlinear to diffusive terms in the Navier–Stokes equations) is large. The number of 3-dimensional degrees of freedom (d.o.f.) increases as $\mathsf{Re}^{9/4}$ as $\mathsf{Re} \to \infty$ in the Kolmogorov 1941 framework (7, §7.4) . For aeronautic flows often $\mathsf{Re} > 10^6$, but for geophysical flows often $\mathsf{Re} \gg 10^8$ (3; 11)

Computations of turbulent flows must contain enough scales to encompass the energy-containing and dissipative scale ranges *distinctly*. Uniform-grid convergence studies on 3D compressible-flow simulations show that in order to achieve the desired scale content, uniform grids must contain at least 2048^3 cells (15). Today such computations can barely be accomplished. A pseudo-spectral Navier-Stokes code on a grid of 4096^3 uniformly spaced points has been run on the Earth Simulator (8), but the Taylor Reynolds number ($\propto \sqrt{\mathsf{Re}}$) is still no more than ≈ 700, very far from what is required for most geophysical flows. The *main goal of our efforts* is to ask, if the significant structures of the flow are indeed sparse, so that their dynamics can be followed accurately even if they are embedded in random noise, then does dynamic adaptivity offer a means for achieving otherwise unattainable large Re values. Thus, we have developed a dynamic geophysical and astrophysical spectral-element adaptive refinement (GASpAR) code for simulating and studying turbulent phenomena.

Several properties of spectral-element methods (SEMs, 1; 13) make them desirable for direct numerical simulation of geophysical turbulence. Perhaps most significant is the fact that SEMs performed at high polynomial degree are inherently minimally diffusive and dispersive. The extent of the spatial and temporal scales that characterize turbulence depends critically on Re, so to draw conclusions we must be certain of this number in our computations. Thus, we cannot allow the numerical methods themselves to introduce diffusion. Also, because SEMs use finite elements, they can be used efficiently in high-resolution turbulence studies in domains with complicated boundaries.

These qualities, together with their good scalability properties (e.g., 6), suggest spectral element methods to be a good basis for high-order adaptive modeling of turbulent flows.

2 How this work supports NCAR strategic priorities

This development work relates directly to at least two of NCAR's strategic priorities: (1) Conducting research in computer science, applied mathematics, statistics, and numerical methods; and (2) Developing and providing advanced services and tools. The connection to both priorities is quite clear, as we are indeed trying to establish the ability of these new high-order adaptive methods to model turbulence, and this has necessitated a good deal of innovation in applied mathematics and numerical methods. Furthermore, the code was released in December 2005, in direct support of priority (2), and, while the next release date has not been set, we will determine this date in early FY2007. See `http://www.cisl.ucar.edu/nar/2006/4i.6.nt.jsp` and `http://www.cisl.ucar.edu/nar/2006/4i.5.ts.jsp` for additional details.

3 2006 accomplishments

FY2006 has seen several improvements to the code. The adaptive refinement algorithms have been tested extensively in the context of linear- and non-linear advection diffusion (14), and we have begun testing the algorithms in the context of Navier–Stokes. In this latter study, we are carrying out a detailed comparison of the dynamics with a pseudo-spectral method, and we are also characterizing the flow using a new multi–resolution analysis technique. We have demonstrated that the algorithms–including adaptive refinement–can capture the behavior of these flows well, thus providing a potential computational savings over non–adaptive methods.

In FY2006, we also began adding an incompressible magnetohydrodynamics (MHD) solver to GASpAR. This is based on the Elsasser (2) formulation of the MHD equations, and the solver currently supports explicit time stepping.

4 2007 plans

If FY2007, we will complete the development and testing of the incompressible MHD solver. The incompressible solvers will be improved by adding optimized preconditioners for their Krylov pressure solver, and by accelerating the inter-processor communications. We plan to add the 'alpha'-model for high Reynolds number calculations in two space dimensions, in either FY2007 or FY2008. Based on our findings regarding the applicability of our high–order adaptive code to incompressible MHD turbulence, we may in FY2007 add to our PDE solver "toolbox" code to accommodate the *compressible* MHD equations, depending on the physics of the objects we want to model (such as in the solar wind and the sun).

5 References

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